Date		

Name _____

Functions Overview Adding Functions Independent Practice

1. Three polynomial functions are given.

$$f(x) = 3x^{2} + 5x - z$$
$$g(x) = 5x^{2} - 3x + z$$
$$m(x) = 2x^{2} - 2x$$

Part A: Write an expression to represent f(x) + [g(x) - m(x)]

Part B: Let p(x) be equivalent to g(x) - f(x). Find p(x) - m(x).

Part C: Write an expression to represent 2[f(x) - g(x)] + 3[m(x)].



2. Two polynomial are given.

$$f(t) = \left(\frac{3}{4}t^3x + 7tx^2 - 10t - x + 1\right)$$
$$g(t) = \left(14tx^2 - \frac{1}{8}t^3x + 6t + x - \frac{5}{2}\right)$$

Part A: Find f(t) + g(t).

Part B: Find f(t) - g(t).

3. Two polynomial are given.

$$h(x) = \left(\frac{1}{3}x^3y - 3x^2y^2 + 9xy^3 - 2\right)$$
$$m(x) = \left(13 - \frac{3}{5}xy^3 + 6x^3y - 2x^2y^2 - 4y\right)$$

Find [h(x) + m(x)] - [h(x) - m(x)].



4. Calisto Publishing Co. sells graphic novels and comics to a large retailer for \$7.50 per book. The daily cost, in dollars, for the company to print x books is C(x) = 4.50x + 1700

Part A: What do the linear term and constant in the cost function represent?

Part B: Write a revenue function and the profit function for Calisto Publishing Co.

5. During the holidays, *Toys 4 U* donate boxes of toys to communities in need. The company ships the toys in three different boxes. The owners need to determine how much more space the larger boxes have for shipping purposes. The volume of the shipping boxes in cubic feet are expressed in the following table.

Size	Volume Function
Small	$s(x) = x^3 + 4x^2 + 11x + 23$
Medium	$m(x) = 2x^3 + 11x^2 + 17x + 6$
Large	$l(x) = 2x^3 + 21x^2 + 38x + 95$

- Part A: Write an expression to represent the extra space available in the medium box when compared to the small box.
- Part B: Write an expression to represent the extra space available in the large box when compared to the medium box.
- Part C: Write an expression to represent the extra space available in the large box when compared to the small box.



6. Donald solved a BEAT THE TEST! problem and gave it to his teacher, Mrs. Calistoga, at the end of the class period. A copy of his work is shown below.

BEAT THE TEST!

1. Consider the following polynomial functions:

 $f(k) = -4k^{4} + 14 + 3k^{2}$ $g(k) = -3k^{4} - 14k^{2} - 8$ $r(k) = -14k^{4} - 2$ $s(k) = -3k^{4} + 5k^{2} - 1$ $t(k) = 4k^{4} + 16k^{2} - 7$

Which of the following represent an expression equivalent to [f(k) + g(k)] - [s(k) - t(k)]?

• -2	$f(\mathbf{k}) + g(\mathbf{k})$	s(k)-t(k)
₿ 0 © 2	$-4k^{4} + 14 + 3k^{2}$	-3k4+5k2-1
$\stackrel{()}{=} \frac{2}{r(k)}$	-3k4 -8 -14k2	-4k ⁴ -16k ² -7
	-7k"+6-11k"	-7k4-11k3-8
	2	
$\left[f(k) + g(k)\right] -$	$\left[s(k) - t(k) \right]$	
r alwid-1	2] - [-] - [+]	
	6-8=-2	

Suppose that Mrs. Calistoga asks you to check and grade Donald's work. Is Donald's work correct? If so, justify each step of his work. If he is not correct, identify the step(s) that are incorrect, and determine the right answer.



Name _____

Date_____

Functions Overview Multiplying Functions Independent Practice

Multiply the following polynomials to write an equivalent expression.

1. $-7x^2y^2(8x^2y - 5xy + 11xy^2 - 31)$

2.
$$\left(\frac{4}{5}t^3 - 5t\right)\left(\frac{1}{4}t^2 - 3t\right)$$

3. $(4x^2 - 7x + 5)(2x - 5)$

4. $(3m^2 - 5m + 2)^2$



Perform the following operations to find the terms to complete the blanks.

5.
$$(a-1)(1 + a + a^2) = -1$$

6. $[(x - y) + 7][(x - y) - 7] = -49$
7. $-5(3y^4 - 4y^2 + 7x)(y^2 - 3x) = (y^2 - 3x)$
 $= -15y^6 + +105x^2$
8. $(3m - 2n)^3 = (9m^2 - 12mn + 4n^2) \cdot = 27m^3 - -8n^3$

Consider the following functions. Perform the operations to determine the values of a, b, c, d, and e.

9. $f(x) = (x^2 + 7x - 12)$ and $g(x) = (x^2 - 9x + 1)$

$$f(x) \cdot g(x) = ax^4 - bx^3 - cx^2 + dx - e$$

10.
$$h(x) = (x^2 + 2)$$
 and $k(x) = (2x^2 - 5x + 7)$
 $h(x) \cdot k(x) = ax^4 - bx^3 - cx^2 + dx - e$



11. Consider the figure below.



What is the total area, in square feet, of the figure?



Functions Overview Using Division to Rewrite Rational Expressions Independent Practice

1. Two polynomial functions are given.

$$a(x) = \frac{x^5 - 1}{x - 1}$$
$$b(x) = \frac{x^6 - 1}{x - 1}$$

Write equivalent expressions for a(x) and b(x) and find the difference of the two expressions.

2. Two polynomial functions are given.

$$m(x) = 2x^3 + x - 16$$
$$p(x) = x^2 - 4$$

Is the degree of the quotient of m(x) and p(x) less than, greater than or equal to the degree of p(x)? Will this always be true when dividing polynomial functions?

3. Write two polynomial functions whose quotient will be the same degree as the divisor.

4. Write two polynomial functions whose quotient will have a degree of zero.



5. An equation is shown.

$$\frac{2x^3 + 4x^2 - 5}{x + 3} = 2x^2 - 2x + 6 + \frac{G(x)}{H(x)}$$

What are the values of G(x) and H(x) that make the equation true?

6. An equation is shown.

$$\frac{3x^3 + 4x^2 + 11}{x^2 - 3x + 2} = Ax^2 + Bx + C + \frac{R(x)}{Q(x)}$$

What are the values of A, B, C, R(x) and Q(x) that make the equation true? Write the most appropriate answer in each space provided.





Functions Overview Using Synthetic Division to Rewrite Rational Expressions Independent Practice

Name _____

1. María Eugenia used synthetic division to solve a *Try It!* Problem from his Algebra 2 Math Nation workbook. Her work is shown below.

Try It!

3. Find the quotient of rational expression below.

$$\frac{5u^{4} + 16u^{3} - 15u^{2} + 8u + 16}{u + 4}$$

$$-4 \int 5 | 6 - 15 8 | 6$$

$$-30 | 6 4 - 16$$

$$5 -4 | 1 | 3 0$$
Anoswer: $5u^{3} - 4u^{3} + u + 13$

$$X = 50008^{1} \cdot 1000$$

4. Find the quotient of the rational expression below.

$$\frac{3x^3 + 2x^2 - 4x + 1}{x - \frac{1}{3}}$$

María had the wrong solution for #3. Identify and correct her error.



2. Use synthetic division to write an equivalent expression for the rational expression below.

$$\frac{12m^3 - 11m^2 + 9m + 18}{m + \frac{3}{4}}$$

3. Use synthetic division to write an equivalent expression for the rational expression below.

$$\frac{2t^3 + 7t^2 - 4t + 7}{t - \frac{1}{2}}$$

4. Use synthetic division to write an equivalent expression for the rational expression below.

$$\frac{4y^3 + 3y^2 - 9y + 2}{y - \frac{1}{4}}$$



5. Which of the following functions would satisfy d(x) such that synthetic division can be performed? Select all that apply.

$$\frac{5x^4 + 3x^3 - 14x^2 - 37x - 9}{d(x)}$$

 $\Box \quad x - \frac{1}{5}$ $\Box \quad x^2 - 5$ $\Box \quad -8 + x$ $\Box \quad 2(x - 1)$ $\Box \quad 5x^{-1} + 1$ $\Box \quad x(5 - x)$ $\Box \quad 4x - 7 + x$

6. The polynomial $2x^3 - 13x^2 + 17x - 10$ represents the volume in cubic feet of a rectangular water holding tank at *Marlon's Farm*. The depth of the tank is represented by the expression (x - 5) feet. The length is 17 feet. Let x represent the width of the tank.

Part A: Write an equivalent expression for the polynomial using the dimensions of the tank.

Part B: Find the width of the tank. (Hint: It's helpful to use synthetic division!)



Functions Overview Composition of Functions Independent Practice

1. Consider $f(x) = x^2 + 3x - 70$ and g(x) = 4(x - 3).

Part A: Find $(f \circ g)(3)$.

Part B: Find g(f(3)).

Part C: Find f(g(-2))

Part D: Find $(g \circ f)(-2)$

2. Given the functions f(x) = 3x and $g(x) = x^2 + 1$, find $[(g \circ f)(x)] - [(f \circ g)(x)]$.



3. Consider the following statement.

"Composition of functions is commutative."

Is this statement always correct? If so, provide an example. If not, provide a counterexample.

4. Consider f(x) = x + 7 and $g(x) = x^2 - 1$.

Part A: Find $(f \circ g)(x)$.

Part B: Find $(g \circ f)(x)$.

5. Consider f(x) = 4x, $g(x) = \sqrt{x^2 - 1}$, and $h(x) = \sqrt{16x - 1}$.

Part A: Is h(x) equivalent to $(f \circ g)(x)$? Justify your answer.

Part B: Is h(x) equivalent to g composed with f of x? Justify your answer.



6. Consider the following functions.

$$f(x) = 7x$$
$$g(x) = \sqrt{x}$$
$$h(x) = x^{2} + 7$$

Match the functions below with their compositions.

____ $H(x) = \sqrt{x^2 + 7}$ A. $(g \circ f)(x)$

$$G(x) = x + 7$$
 B. $(h \circ g)(x)$

$$F(x) = 7x^2 + 49$$
 C. $(f \circ g)(x)$

$$H(x) = 49x^2 + 7$$
 D. $(g \circ h)(x)$

$$G(x) = 7\sqrt{x}$$
 E. $(h \circ f)(x)$

$$F(x) = \sqrt{7x} \qquad \qquad F. \qquad (f \circ h)(x)$$



Date_____

Functions Overview Inverse Functions - Part 1 Independent Practice

1. Determine whether each function is a one-to-one function. If it is one-to-one, then write the inverse function.

Part A: $\{(7, 9), (13, 6), (3, 9), (11, -4), (1, 7)\}$

Part B: {(11, 8), (5, 6), (3, 7), (21, 2), (2, 5)}

2. Find the inverse of the following functions.

Part A: $f(x) = \frac{x-8}{5}$

Part B:
$$f(x) = \sqrt[3]{x+3}$$

Part C:
$$f(x) = \frac{1}{x-7}, x \neq 7$$

Part D: $f(x) = \frac{x+1}{1}$



3. Graph the inverse of each function on the same coordinate plane. Is the function one-to-one?

Part A:









4. MT&A uses the function, f(x) = 475 - 15x, to depreciate smart phones, where f represents the value of a smart phone and x represents the number of months since its purchase.

Part A: Find f^{-1} , and explain what it represents in this situation.

Part B: When will the depreciated value of a smart phone be less than \$100?

Part C: What does x represent in $f^{-1}(x) = 30$? What is the value of x?

Part D: Graph f(x) and $f^{-1}(x)$ on the same coordinate plane below.





Functions Overview Inverse Functions – Part 2 Independent Practice

1. Consider the function f(x) = 7x - 4.

Part A: Find $f^{-1}(x)$

Part B: Find $f(f^{-1}(x))$

Part C: Find $f^{-1}(f(5))$

Part D: Find $f(f^{-1}(\mathbf{0}))$

2. If
$$f(x) = 4x + 3$$
, show that $f^{-1}(x) = \frac{x-3}{4}$.

3. Is the function $f(x) = x^2 - 9$ an invertible function? If not, restrict the domain so that f(x) is an invertible function.



4. If $f(x) = x^3 - 7$, show that $f^{-1}(x) = \sqrt[3]{x+7}$.

5. If
$$f(x) = \frac{x-6}{11}$$
, show that $f^{-1}(x) = 11x + 6$.

6. A quadratic function is shown.



Restrict the domain of f(x) so that $f^{-1}(x)$, the inverse of f(x), is a function.

7. Two functions are given.

$$f(x) = 3x + 6$$
 $g(x) = \frac{1}{3}x - 3$

Use composition of functions to determine if f(x) and g(x) are inverses.



8. Two functions are given.

$$f(x) = -3x + 3$$
 $g(x) = -\frac{1}{3}x + 1$

Use composition of functions to determine if f(x) and g(x) are inverses.

9. In the Independent Practice for Topic 5, you were asked the following question.

Consider the following statement.

"Composition of functions is commutative."

Is this statement always correct? If so, provide an example. If not, provide a counterexample.

Part A: Is your first answer really the best or most precise answer or should you change your answer based on the skills learned in Topics 6 and 7? Justify your answer.

Part B: Circle the best answer that completes the following statement.

Composition of functions is always | sometimes | never commutative.



Date	
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Functions Overview Recognizing Even and Odd Functions Independent Practice

Function	Even	Odd	Neither
$f(x) = \frac{ x }{x^2 + 1}$			
$g(x) = x(x^4 + 7)$			
h(x) = 0			
$j(x) = x^{-3}$			
	•		

1. Determine if the following functions are even, odd, or neither.

2. Determine the values of k, m, and n that creates an odd function for $p(x) = x^k - x^m + n$. Justify your answer.

A k = 4, m = 2, and n = 1

- B k = 3, m = 1, and n = 9C k = 7, m = 5, and n = 2
- C = 7, m = 5, and n = 2
- D None of the combinations above will create an odd function for p(x).
- 3. Complete the statements.
 - > If a function is even, then $f(-x) = _$ and has symmetry about the _____.
 - > If a function is odd, then f(-x) =_____ and has symmetry about the ______.



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4. Consider the following graphs. Label each graph as even, odd, or neither in the space provided.





5. Sketch an example of a polynomial function that is an even function below.



6. Sketch an example of a polynomial function that is an odd function below.





Functions Overview Key Features of Graphs of Functions Independent Practice

1. Complete the blanks with the word bank provided (some words may be used more than once).

Increasing	y –intercepts	Dependent	x
Independent	f(x) > 0	Input	Output
у	Decreasing	f(x) < 0	x –intercepts

- Relative maximum: the point on a graph where the interval changes from _______ to ______.
- Solutions, Zeros, or Roots: the values for which the function equals zero. These are also the ______ of the graph.
- > Increasing intervals: as the x –values _____, the y values _____.

> Positive intervals: intervals of a function f(x) over which _____.

- > Decreasing intervals: as the x –values _____, the y values
- > Range: the _____ or the y –values.
- Relative minimum: the point on a graph where the interval changes from to _______.
- > Negative intervals: intervals of a function f(x) over which ______.
- > Domain: the input or the _____ values.

_____·

2. Determine the domain and range for $t(x) = -(x + 10)^2 - 64$



3. Consider the following graph of r(x).



Which of the following statements are false of the function? Select all that apply.

- \Box The function has solutions at (-6.1, 0), (0, 0), and (1, 5).
- \Box The function has only one positive interval ($-\infty$, -6.1).
- \Box The function is decreasing over the intervals (-7, -5) and (1, 7).
- \Box The domain of the function is $(-\infty, 7)$.
- $\hfill\square$ The function has two relative minimums.
- \Box The range of the function is $[-8, \infty)$.
- 4. Determine the following for the equation $y = \sqrt{x-1}$.

Interval(s) Increasing:

Interval(s) Decreasing:

Positive Interval(s):

Negative Interval(s):

Domain:

Range:

y –intercept:

x -intercept(s):



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5. Consider the following graph of m(x) and determine the following answers in interval notation.



Interval(s) Increasing:

Interval(s) Decreasing:

Positive Interval(s):

Negative Interval(s):

Domain:

Range:

y-intercept

x - intercept(s)



6. The cost of an air conditioner is \$130. The cost to run the air conditioner is \$0.25 per minute for the first 400 minutes. Write the function that models the total cost of running the air conditioner for the first 400 minutes and determine the key features of the function.

Function:

Domain:

Range:

y –intercept:

x -intercept(s):

Circle: Increasing | Decreasing



Name _____

Functions Overview Transformation of Functions – Part 1 Independent Practice

1. Determine whether the following transformations are on the independent or dependent variable and describe the transformation.

$$c(x) = f(x) + 7$$
 $g(x) = f(x) - 11$

$$h(x) = f(x + 3)$$
 $j(x) = f(x - 4) + 9$

$$m(x) = \frac{3}{4}f(x) \qquad \qquad n(x) = 4f(x)$$

$$p(x) = f(3x) \qquad \qquad r(x) = f\left(\frac{1}{3}x\right)$$

$$v(x) = -\frac{1}{2}f(x)$$
 $w(x) = f(-2x)$

2. Let $f(x) = \sqrt{x}$, where $x \ge 0$. Write a function that represents the following transformations on f(x).

Part A: A translation left seven units and up three units.

Part B: A stretch by any factor that reflect f over the y-axis and a translation down one unit.



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3. Tristan was asked to explain the transformation of g(x) from f(x) where g(x) = f(x + 5). Tristan said it shifts 5 units to the right. His best friend, Cielomar, explained to him that the dependent variable is not being transformed so in order to get the same output, we must choose values for x that are 5 units to the left. Complete the table below to justify Cielomar's reasoning.



4. Consider the functions $f(x) = x^2$, g(x) = f(x-2), h(x) = f(2x), and $p(x) = \frac{1}{2}f(x)$. You can use the coordinate plane below to help guide your solution process.

Part A: How do the values of h and p relate to the values of f?

Part B: There is a point (x + 2, g(x + 2)) on g(x). Use this point to prove that the graph of g is the graph of f translated 2 units to the right.





5. Consider the following table that models an exponential function.

x	у
-1	$\frac{1}{2}$
0	1
1	2
2	4
3	8

Complete the following table for the transformation g(x) = f(x - 1).

x	f(x-1)	f(x-1)	g(x)
		f(-1)	$\frac{1}{2}$
		<i>f</i> (0)	1
		<i>f</i> (1)	2
		f(2)	4
		f(3)	8

Functions Overview Transformation of Functions – Part 2 Independent Practice

1. Consider the functions $f(x) = 3^x + 2$ and $g(x) = -f(\frac{1}{3}x - 3) + 2$. Describe the transformation on f(x).

2. The table below represents p(x).

x	p(x)
-3	11
0	2
1	3
4	18

Complete the following table for the transformation $g(x) = 2p\left(\frac{1}{2}x\right) - 1$.

x	$g(x) = -p\left(\frac{1}{2}x\right) - 1$	g(x)



3. Consider $f(x) = x^2$ and $g(x) = f\left(\frac{1}{2}x\right) - 1$.

Part A: Graph each function in the coordinate plane below.



- Part B: Write the formula of a quadratic function whose graph would be a vertical stretch of the graph of g.
- 4. Suppose you own a publishing company that sells political non-fiction books to a large retailer for \$8.25 per book. The daily cost of the company to print x books is C(x) = \$8.25x + 1500 dollars. If the marginal cost of producing books doubles but the fixed cost decreased by 30%, then the new cost function will be represented by function K(x). Write K(x) as a transformation of C(x).



x	f(x)	$\boldsymbol{g}(\boldsymbol{x})$	h(x)	m(x)
—7	-3	-6	-4	-17
-2	2	4	1	-2
3	7	14	6	13
8	12	24	11	28
13	17	34	16	43

5. Consider the following table where g(x), h(x), and m(x) are transformations on f(x).

Write g(x), h(x), and m(x) as transformations of f(x).

6. The graphs of p(x) and v(x) are shown below.



Part A: Write p(x) as a transformation of v(x).

Part B: Write v(x) as a transformation of p(x).



7. The function f(x) is shown below.



The function is transformed to create the function g(x) such that $g(x) = 2f(\frac{1}{2}x) + 1$. Complete the table below.

x	x-coordinate	y-coordinate
Α'		
Β'		
С′		
<i>D'</i>		
Ε'		



Functions Overview Average Rate of Change of Functions Independent Practice

1. Consider the function $f(x) = 0.3x^2 - 2x + 6$. The graph of f(x) is shown below.



Determine the average rate of change of f(x) for the interval [-1, -2]. Round your answer to the nearest tenth.

2. A retail store is having a back-to-school sale on chromebooks. At 10:00AM, the store has sold 8 chromebooks. At 7:00PM, the store discovers that it has sold 32 chromebooks. What is the average rate of change between 10:00AM and 7:00PM? Round your answer to the nearest unit.



3. The following table represents a square root function.

x	g(x)
1	2
9	6
25	10
49	14
81	18

Part A: Determine the difference between the rate of change of g(x) over the interval [9,49] and the rate of change of g(x) over the interval [25,81]. Round your answer to the nearest hundredth.

Part B: Which of the following values represent the rate of change over the interval [121, 225]?

 $\begin{array}{c} A & \frac{1}{13} \\ B & \frac{1}{11} \\ C & \frac{2}{13} \\ D & \frac{2}{11} \end{array}$



4. For the function $f(x) = 2^x + 3$, Vidal calculated the rate of change of f(x) over the interval [-2, 7] rounded to the nearest hundredth. His work is shown below.

We have that a = -2, b = 7, $f(x) = 2^x + 3$.

Thus,
$$\frac{f(b) - f(a)}{b - a} = \frac{2^{(7)} + 3 - (2^{(-2)} + 3)}{7 - (-2)} = \frac{511}{36}.$$

Determine if Vidal's answer is correct explaining the steps he followed and identifying errors, if any.

5. The average rate of change of h(x) over the interval [a, b] is zero and b > a.

Part A: What can we determine about h(x) over the interval?

- A h(x) is increasing over the interval.
- **B** h(x) is constant over the interval.
- C h(x) is odd over the interval.
- **D** h(x) on interval [a, b] has h(a) = h(b)

Part B: Justify your answer from Part A with an example.

